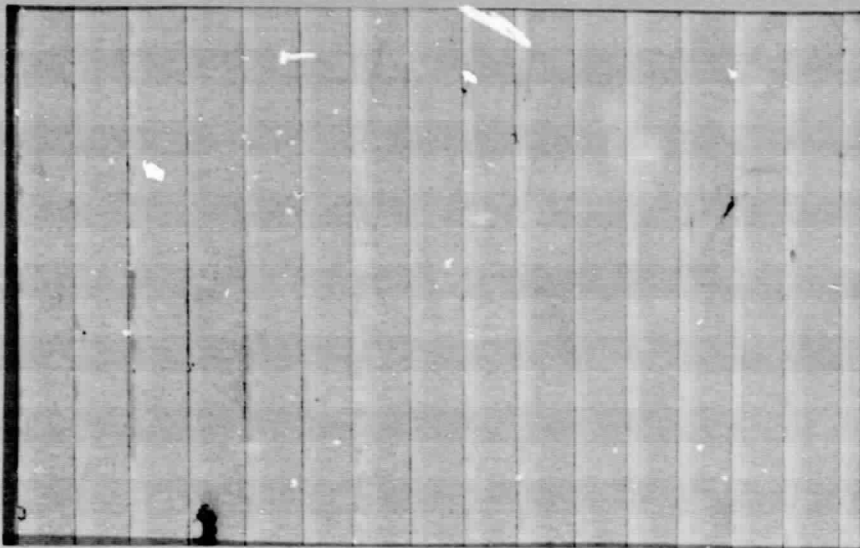


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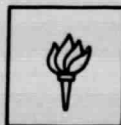


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**A SCHEME FOR COMPUTING SURFACE LAYER TURBULENT
FLUXES FROM MEAN FLOW "SURFACE OBSERVATIONS"**

Martin I. Hoffert

and

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A SCHEME FOR COMPUTING SURFACE LAYER TURBULENT FLUXES
FROM MEAN FLOW "SURFACE OBSERVATIONS"

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Abstract. A physical model and computational scheme are developed for generating turbulent surface stress, sensible heat flux and humidity flux from mean velocity, temperature and humidity at some fixed height in the atmospheric surface layer, where conditions at this reference level are presumed known from observations or the evolving state of a numerical atmospheric circulation model. The method is based on coupling the Monin-Obukov surface layer similarity profiles which include buoyant stability effects on mean velocity, temperature and humidity to a "force-restore" formulation for the evolution of surface soil temperature to yield the local values of shear stress, heat flux and surface temperature. A self-contained formulation is presented including parameterizations for solar and infrared radiant fluxes at the surface.

In addition to reference-level mean flow properties parameters needed to implement the scheme are the thermal heat capacity of the soil per unit surface area, surface aerodynamic roughness, latitude, solar declination, surface albedo, surface emissivity and atmospheric transmissivity to solar

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radiation.

Sample calculations are presented for a case with constant atmospheric forcing at the reference level and for a variable atmospheric forcing case at conditions corresponding to Kahle's (1977) measurements of windspeed and air temperature and radiometer soil surface temperature measurements under dry vegetatively sparse conditions in the Mohave desert in California, USA. The latter case recovered the observed ground temperature variation over a diurnal cycle reasonably well for the parameters used, and displayed a variety of buoyant stratification conditions which can occur in atmospheric surface layers including convectively unstable, stable and stable-decoupled zones.

1. Introduction

For a number of applications in micrometeorology and air pollution transport analysis it is desired to know the turbulent shear stress τ and the turbulent heat flux H in the so-called constant flux atmospheric surface layer immediately above the earth's surface when direct measurements of the relevant turbulent fluctuation moments are unavailable. What is often available is information on mean horizontal velocity, temperature and humidity at some reference level z_1 in the surface layer where a time series has been measured by appropriate instrumentation. Generally the reference-level mean velocity and temperature $\overline{u}_1(t)$ and $\overline{T}_1(t)$ are time-dependent over diurnal cycles, where the overbar average is understood in the usual sense to denote averaging over short period turbulent fluctuations. The problem to be considered here is the generation of consistent values of $\tau(t)$ and $H(t)$ from the reference-level mean flow using a physical model for the structure of the atmospheric surface layer and the underlying layer of ground.

An important consideration in this context is that the aerodynamic drag coefficient $C_D \equiv \tau/(\rho u_1^2)$ and the heat transfer coefficient $C_H \equiv -F/(\rho C_p u_1 (T_1 - T_g))$, where ρ is the air density, C_p is the constant-pressure specific heat of air and T_g is the surface temperature (overbars are dropped here and henceforth on reference-level properties), are not constant but can depend strongly on the buoyant stratification of the surface layer. Generally both C_D and C_H increase for unstable (heated from below) conditions and decrease for stable (cooled from below) conditions. Under extremely stable conditions, which are nevertheless encountered in practice, the surface layer turbulence can be extinguished entirely, leading to decoupling of the surface layer from a possibly turbulent zone some distance above it. Under these conditions the drag and heat transport drop substantially to zero, laminar transport being negligible in the context of air-surface interactions.

The structure of the surface layer has been extensively treated in recent years in the context of Monin-Obukhov similarity theory in which the horizontal velocity and temperature profiles $\bar{u}(z)$ and $\bar{T}(z)$ are uniquely defined by the aerodynamic roughness length z_o , the friction velocity $u^* \equiv (\tau/\rho)^{1/2}$, the buoyant temperature scale $T^* \equiv -F/(\rho C_p u^*)$ and the surface temperature $T_g \equiv \bar{T}(z_o)$ (Monin and Yaglom, 1971, p.430). In particular, the Monin-Obukhov length scale $L \equiv T_g u^{*2}/(\kappa g T^*)$, where κ is von Kármán's constant and g is the gravitational acceleration, is defined with dimensionless mean velocity and temperature gradients in the surface layer given by the "universal" functions $\phi_m(z/L) \equiv (\kappa z/u^*) \partial \bar{u} / \partial z$ and $\phi_H(z/L) \equiv (\kappa z/T^*) \partial \bar{T} / \partial z$. In practice, the ϕ_m and ϕ_H functions are found experimentally and the velocity and temperature profiles determined by integration of the gradient functions from $z = z_o$ to some arbitrary point in the surface layer. In developing the details of the present scheme, the functional forms of $\phi_m(z/L)$ and $\phi_H(z/L)$ resulting

from an experiment performed in wheat fields of Kansas, USA, by Businger *et al.* (1971) are used. It is recognized that other workers have measured different functional forms, for example Hicks (1976) and Sisterson and Frenzen (1978) whose results differ primarily in the stable case. In addition Hicks (1976) has questioned the inequality between the neutral values of ϕ_m and ϕ_H in the Businger *et al.* functions which he finds difficult to correlate with the physical processes involved.

We hasten to point out at the outset that other forms of the Monin-Obukhov functions can be used in the framework of the scheme developed here as experimental discrepancies are ultimately resolved. Also, we should, strictly speaking be working with potential, rather than physical, temperature, but for surface-layer reference levels of the order of ten meters or less, the results should be insignificantly affected. Humidity, on the other hand, can significantly affect the computed fluxes, and its effects are included in the subsequent formulation; it is omitted here only to more rapidly focus on the central problem. Namely, given the aerodynamic roughness z_0 , the reference-level velocity and temperature u_1 and T_1 and the surface temperature T_g , the flux parameters u^* and T^* are uniquely defined by the Monin-Obukhov similarity profiles; however, while z_0 can often be characterized by the known properties of the terrain in question, and u_1 and T_1 are known, the surface temperature T_g is almost never measured on a routine basis. And even if the soil surface temperature immediately below the reference-level instruments were known, its significance might be unclear since it could well represent only the characteristics of the immediate surface type, rather than the regional average of interest for flux estimations.

A more useful approach in practical situations is to characterize the thermal and radiant properties of the soil surface layer and to compute the

soil temperature simultaneously with the Monin-Obukhov parameters at each timestep by solving the soil heat conduction equation under the influence of solar and infrared radiation at the surface as well as the turbulent sensible and latent heat flux associated with the atmospheric surface layer. A number of approaches to the calculation of ground temperature exist in the literature, primarily in relation to parameterization of fluxes in general circulation models, but in all of this published work the atmospheric fluxes are represented by constant values of the drag and heat transfer coefficients. Perhaps the most accurate treatments of the ground temperature unsteady heat conduction equation are the finite-difference solutions of Benoit (1976) and Kahle (1977). In the interest of conserving computer time a number of integral approaches leading to ordinary differential equation models for surface soil temperature have been proposed in recent years. A method developed independently by Arakawa (1972) and by the British Meteorological Service (Corby *et al.*, 1972; Rountree, 1975) utilizes a rate equation for a ground "slab" temperature T_g dependent upon forcing by the sum of atmospheric and radiant energy fluxes; however, as noted by Bhumralkar (1975) this method omits the influence of soil heat flux on the underside of the integral slab. Moreover, the temperature being predicted is not truly the surface temperature consistent with the similarity atmospheric surface layer, but some depth-average of the temperature in the thermally active layer of soil beneath the surface.

The scheme to be developed here is based on the so-called "force-restore" ordinary differential rate equation independently proposed by Bhumralkar (1975) and Blackadar (1976) and recently evaluated in comparison with a 12-layer finite-difference solution to the partial differential heat conduction equation by Deardorff (1978). The force-restore formulation has the advantage of predict-

ing the soil temperature T_g at the surface, rather than a depth-aveaged value, and contains a mechanism by which a deeper soil layer can influence the surface temperature. Deardorff (1978) found the force-restore formulation was computationally more efficient than the use of multiple soil layers and superior to five other approximate methods in current use when diurnal forcing was present. In a related study, Deardorff (1977) has proposed an analogous force-restore formulation for the soil-surface moisture fraction as a parameterization of ground-surface moisture content for use in atmospheric prediction models.

In what follows, the relevant aspects of Monin-Obukhov similarity theory are developed, the force-restore equation for soil surface temperature is derived, expressions are given for the solar and infrared radiation forcing terms, and the coupled system computer code is described. Subsequently, computational results are presented for both constant and variable atmospheric forcing and comparisons made with observational data.

2. Monin-Obukhov Profiles in the Atmospheric Surface Layer

The planetary boundary layer (PBL) is the region of the lower atmosphere where flow is turbulent on a microscale by virtue of shear- and convection-induced turbulent fluctuations associated with the underlying solid or water surface. In the lower PBL, i.e. the so-called surface layer where the turbulent shear stress, turbulent sensible heat flux and turbulent humidity flux may be treated as constant, the relevant flow variables are velocity $(u, v, w) = (\bar{u} + u', v', w')$, temperature $T = \bar{T} + T'$ and specific humidity $q = \bar{q} + q'$, where overbars denote Reynolds averages and "primes" denote fluctuations. We assume for the present problem that the known reference-level properties u_1 , T_1 and q_1 represent \bar{u} , \bar{T} and \bar{q} at some height z_1 within the surface layer.

What we would like to know however are the Reynolds stress $\tau \approx -\rho \overline{u'w'}$, sensible heat flux $F = \rho C_p \overline{w'T'}$ and evaporative flux $E = \rho \overline{w'q'}$ which may be considered constant in the surface layer. The mean air density ρ may also be treated as constant at its surface value ($\approx 1.23 \text{ kg/m}^3$), but fluctuations in density are important in buoyancy-generated (convective) turbulence. It follows from the equation of state $\rho = p/[R(1 + 0.61q)]T$ that the effects of water vapor variations on density can be handled by working with the *virtual* temperature $T_v \equiv (1 + 0.61q)T$. Accordingly, the Boussinesq approximation (which neglects pressure fluctuations) for air with some water vapor relates density fluctuations to fluctuations in virtual temperature, $\rho'/\rho \approx -T_v'/T_v$. Ordinarily we can take $T \approx T_v$ *except* when differences or fluctuations are involved. An important quantity in buoyantly-interactive turbulent flows is the buoyancy flux $F_b \equiv -C_p \overline{T' \rho' w'} \approx \rho C_p \overline{w'T_v'} \approx F + 0.61 C_p \overline{T'E}$, where E , F and F_b are all positive upwards, and buoyantly unstable, neutral and stable conditions correspond to F_b negative, zero and positive, respectively.

To relate the mean flow measurements in the surface layer to the surface layer fluxes we shall make use of the similarity theory originally proposed by Monin and Obukhov and subsequently developed by many others. It is convenient here to introduce turbulent velocity, thermal and humidity scales related to the surface fluxes as follows,

$$u^* \equiv (\tau/\rho)^{1/2}, \quad T^* \equiv -F/(\rho C_p u^*), \quad q^* \equiv -E/(\rho u^*). \quad (2)$$

In addition, the buoyant fluctuation thermal scale

$$T_v^* \equiv -F_b/(\rho C_p u^*) = T^* + 0.61 \overline{T} q^* \quad (3)$$

plays an important role in Monin-Obukhov theory. The fundamental assumption based on dimensional analysis arguments is that vertical gradients of mean

flow properties in the surface layer may be expressed,

$$\frac{\partial \bar{u}}{\partial z} = \frac{u^* \phi_m(z/L)}{\kappa z}, \quad \frac{\partial \bar{T}}{\partial z} = \frac{T^* \phi_H(z/L)}{\kappa z}, \quad \frac{\partial \bar{q}}{\partial z} = \frac{q^* \phi_q(z/L)}{\kappa z}, \quad (4)$$

where the coefficient κ known as von Kármán's constant has a numerical value of $\kappa \approx 0.35$ (Businger *et al.*, 1971),

$$L \equiv T_g u^{*2} / (\kappa g T_v^*) \quad (5)$$

is a lengthscale characterizing buoyancy effects on turbulence known as the Monin-Obukhov length, and ϕ_m , ϕ_H and ϕ_q are "universal" functions of the surface layer stability variable $\zeta = z/L$. In general L is negative for unstable stratification, positive for stable stratification and approaches positive or negative infinity under neutral conditions. Businger *et al.* (1971) have measured and curve fit the functions $\phi_m(\zeta)$ and $\phi_H(\zeta)$ under both unstable and stable conditions. They find

$$\phi_m(\zeta) = (1 - 15\zeta)^{-1/4}, \quad \phi_H(\zeta) = \alpha_0 (1 - 9\zeta)^{-1/2}$$

under unstable ($\zeta < 0$) conditions and

$$\phi_m(\zeta) = 1 + 4.7\zeta, \quad \phi_H(\zeta) = \alpha_0 + 4.7\zeta$$

under stable ($\zeta > 0$) conditions, where the turbulent Prandtl number $\alpha = \phi_H/\phi_m$ has the numerical value at neutral stability of $\alpha_0 = 0.74$. More generally, of course α is itself a function of ζ . On the other hand Dyer (1967) has shown $\phi_q/\phi_H \approx 1$ under a range of unstable conditions. Accordingly, we shall take $\phi_q(\zeta) = \phi_H(\zeta)$ in the following analysis.

Now, integrating the relations in (4) between the "surface", located at aerodynamic roughness height z_0 where $\bar{u}(z_0) = 0$, $\bar{T}(z_0) = T_g$ and $\bar{q}(z_0) = q_g$ by definition, and some arbitrary height z_1 in the surface layer, gives the Monin-Obukhov profiles for turbulent-mean velocity, temperature and humidity:

$$\bar{u}(z_1) = (u^*/\kappa)[\ln(z_1/z_0) - \psi_u(z_1/L)], \quad (6)$$

$$\bar{T}(z_1) = T_s + [T^*(\alpha_0/\kappa)][\ln(z_1/z_0) - \psi_T(z_1/L)], \quad (7)$$

$$\bar{q}(z_1) = q_s + [q^*(\alpha_0/\kappa)][\ln(z_1/z_0) - \psi_T(z_1/L)], \quad (8)$$

where the stability-dependent profile functions ψ_u and ψ_T are defined formally by the integrals,

$$\psi_u(z_1/L) \equiv \int_{z_0}^{z_1} \frac{[1-\phi_m(\zeta)]d\zeta}{\zeta} = \int_0^{z_1} \frac{[1-\phi_m(\zeta)]d\zeta}{\zeta},$$

$$\psi_T(z_1/L) \equiv \int_{z_0}^{z_1} \frac{[1-\alpha_0^{-1}\phi_H(\zeta)]d\zeta}{\zeta} = \int_0^{z_1} \frac{[1-\alpha_0^{-1}\phi_H(\zeta)]d\zeta}{\zeta}.$$

These profile functions may be obtained explicitly, for example, by substituting the $\phi_m(\zeta)$ and $\phi_H(\zeta)$ expressions of Businger *et al.* given previously and integrating. The result (obtained after some algebra) is summarized in Table 1. The mean profile functions (6)-(8) with the stability-dependent

Table 1. Stability-dependent functions in Monin-Obukhov profiles.

stability variable	$\psi_u(z_1/L)$	$\psi_T(z_1/L)$
$(z_1/L) < 0$ [unstable]	$2\ln\left(\frac{1+[1-15(z_1/L)]^{1/4}}{2}\right)$ $+ \ln\left(\frac{1+[1-15(z_1/L)]^{1/2}}{2}\right)$ $+ 2\tan^{-1}[1-15(z_1/L)]^{1/4} + \pi/2$	$2\ln\left(\frac{1+[1-9(z_1/L)]^{1/2}}{2}\right)$
$(z_1/L) > 0$ [stable]	$- 4.7(z_1/L)$	$- 6.4(z_1/L)$

functions of Table 1 are substantially the same as the surface layer profiles used by Deardorff (1972) in his PBL parameterization scheme.

Now, assuming these stability-dependent profiles describe the vertical variations of velocity, temperature and humidity in the surface layer, we may apply them to the present problem of finding the surface layer fluxes as follows. Note first from (3) and (5) that the Monin-Obukhov similarity length scale can be written

$$L = \frac{u_*^2 T_g}{\kappa g (T^* + 0.61 T_g q^*)}$$

where the gravitational acceleration g is approximately 9.81 m/s^2 at sea level. Using equations (6)-(8) to express the turbulent velocity, temperature and humidity scales u^* , T^* and q^* , and substituting these into the above leads to the following implicit equation for Monin-Obukhov length:

$$f(L) \equiv L - \frac{\alpha_0 u_1^2 T_g}{g [T_1 - T_g + 0.61 T_g (q_1 - q_g)]} \cdot \frac{[\ln(z_1/z_0) - \psi_T(z_1/L)]}{[\ln(z_1/z_0) - \psi_u(z_1/L)]^2} = 0. \quad (9)$$

Accordingly, if the reference-level velocity $u_1 = \bar{u}(z_1)$, temperature $T_1 = \bar{T}(z_1)$, and humidity $q_1 = \bar{q}(z_1)$ are known, for example from surface observations, the aerodynamic roughness z_0 is specified, for example as a function of the underlying terrain, and if the "surface" temperature $T_g = \bar{T}(z_0)$ and humidity $q_g = \bar{q}(z_0)$ at the ground/air interface are known, for example from a simultaneous heat transfer analysis as described later in this report, then the Monin-Obukhov length L is given in principle by the solution to (9). For the unstable ($L < 0$) case the $\psi_u(z_1/L)$ and $\psi_T(z_1/L)$ functions are transcendental (Table 1) and an explicit solution for L is not feasible. However, experience has shown that a straightforward Newton-Raphson iteration to find the root of $f(L) = 0$ is generally effective in the unstable case provided the initially

guessed value of L has the proper (negative) sign.

Under stable ($L > 0$) conditions the profile functions are much simpler, $\psi_u(z_1/L) = -4.7z_1/L$ and $\psi_T = -6.4z_1/L$ (Table 1), and equation (9) reduces to a quadratic in L ,

$$aL^2 + bL + c = 0,$$

where $a = [\ln(z_1/z_0)]^2$, $b = \ln(z_1/z_0)(9.4 - \alpha_0 u_1^2 T_g / \{g[T_1 - T_g + 0.61T_g(q_1 - q_g)]\})$ and $c = 4.7z_1(4.7z_1 - u_1^2 T_g / \{g[T_1 - T_g + 0.61T_g(q_1 - q_g)]\})$. Notice here that we can identify the quantity $T_1 - T_g + 0.61T_g(q_1 - q_g)$ with the difference in virtual temperature between the reference height and the surface $T_{v1} - T_{vg}$. This is to be expected since virtual temperature differences are related to buoyancy differences in humid air. The solution to the quadratic takes the usual form,

$$L = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

for the stable ($L > 0$) case, although the nonphysical negative root has been discarded. For a real positive root to exist we need $4ac < 0$. Since the coefficient a is always positive [being the square of $\ln(z_1/z_0)$], this means c must be negative, that is: $4.7z_1 - u_1^2 T_g / [g(T_{v1} - T_{vg})] < 0$. This condition may be written in the form

$$0 < \frac{gz_1(T_{v1} - T_{vg})}{T_g u_1^2} < 0.21$$

which is basically a condition on a finite-difference form of the Richardson number $Ri \equiv (g\partial T_v / \partial z) / [T_g (\partial u / \partial z)^2]$ corresponding physically to the requirement that the Richardson number must be below some critical value $Ri < Ri_{cr} = 0.21$ for turbulent flow to exist under stably stratified conditions. In practice this means surface and reference-level properties must correspond to Richard-

son numbers below 0.21 for a real positive Monin-Obukhov length to exist and be calculable for the stable case. Generally speaking, a Richardson number above 0.21 computed in this way would indicate a reference level above the presumably shallow surface layer, and an associated decoupling of the surface turbulent zone from mixing zones above.

To summarize, given the aerodynamic roughness z_0 , the reference-level measurements u_1 , T_1 and q_1 and the surface temperature and humidity T_s and q_s , we can find the Monin-Obukhov length L from equation (9). Generally, we would expect L to vary markedly with time-of-day, with positive stable values characterizing the nocturnal ground-based inversion, a transition through an infinite neutral condition near sunrise, and a subsequent evolution of negative unstable values during the day with a negative minimum in mid-afternoon corresponding to peak ground temperatures, followed by a transition back to positive stable values near sunset. Knowing L , the frictional velocity, temperature, and humidity scales can be found from

$$u^* = \kappa u_1 / [\ln(z_1/z_0) - \psi_u(z_1/L)], \quad (10)$$

$$T^* = \kappa (T_1 - T_s) / [\alpha_0 [\ln(z_1/z_0) - \psi_T(z_1/L)]], \quad (11)$$

$$q^* = \kappa (q_1 - q_s) / [\alpha_0 [\ln(z_1/z_0) - \psi_T(z_1/L)]] = T^* (q_1 - q_s) / (T_1 - T_s), \quad (12)$$

which follow immediately from equations (6)-(8).

3. Surface Soil Temperature from Force-Restore Rate Equation

Since T_s and q_s are not specified by the surface observations, it is necessary to introduce some additional considerations to evaluate them. As discussed in the Introduction, our approach to computing T_s and q_s at each timestep is to use the force-restore formulation for the soil surface temperature (Bhumralkar, 1975; Blackadar, 1976; Deardorff, 1978) as developed below.

Strictly speaking T_g is the air temperature at the z_0 level of the atmospheric surface layer. We shall assume however, that air at this level is in thermal equilibrium with the underlying solid or liquid surface, neglecting any possible effects associated with intermediate layers. Accordingly, we can identify T_g with the ground (or soil) temperature at the surface. To a good approximation this temperature is governed by the one-dimensional heat conduction equation for a semi-infinite slab heated or cooled at the surface by radiation and the turbulent sensible and latent heat fluxes. By the Fourier heat conduction law the rate of heat flow vertically through the soil at depth z below the surface is proportional to the temperature gradient,

$$G(z, t) = - \lambda \partial T / \partial z,$$

where λ is the coefficient of thermal conductivity; a function in general of soil type and water content. A typical value is $\lambda = 2.5 \times 10^{-3}$ cal/ $^{\circ}$ K \cdot cm \cdot s (≈ 1.05 J/ $^{\circ}$ K \cdot m \cdot s) for soil, although Fig. 37 of Sellers (1965) indicates variations of a factor of two or more from this value are certainly possible depending on soil type and water content.

Since the soil surface temperature varies largely in response to diurnal cycles of solar radiation we need to consider the unsteady form of the soil heat conduction equation,

$$\rho c \frac{\partial T}{\partial t} = - \frac{\partial Q}{\partial z} = \lambda \frac{\partial^2 T}{\partial z^2}, \quad (13)$$

where ρ is the soil density, c is the soil specific heat per unit mass. Clearly then, the product ρc represents the soil's specific heat per unit volume. The quantity $K \equiv \lambda / (\rho c)$ may be defined as the thermal diffusivity. Here a representative value for soil is $K = 5 \times 10^{-3}$ cm 2 /s (5×10^{-7} m 2 /s), although again

Sellers (1965, Fig. 39) indicates factor-of-two or more variations in K are possible depending on soil composition and water content. Accordingly, a typical specific heat per unit volume of soil is $\rho c = \lambda/K = 0.5 \text{ cal/}^\circ\text{K}\cdot\text{cm}^3$ ($\approx 2.1 \times 10^6 \text{ J/}^\circ\text{K}\cdot\text{m}^3$).

Noting that the vertical coordinate in equation (13) is positive downward, the relevant boundary conditions on (13) for the present problem are

$$G(0,t) = -\lambda(\partial T/\partial z)_0 = +S - R - F - LE, \quad (14a)$$

$$G(\infty) = 0, \quad (14b)$$

where S is the net flux of solar radiation absorbed by the ground surface, R is the net longwave flux radiated up into the atmosphere, F is the sensible heat flux and LE is the latent heat flux leaving the surface by turbulent transport vertically through the atmospheric surface layer. In the general case the net heat flux to the ground at the surface $G(0,t)$ is too complicated a function of the time t to admit a simple, analytic solution to (13) with these boundary conditions. We can recognize however that surface temperature variations over timescales of, say, a few days are driven largely by the diurnal solar radiation cycle of frequency $\Omega = 2\pi \text{ radians/day} = 7.27 \times 10^{-5} \text{ s}^{-1}$. It is instructive therefore to consider the response of (13) to a periodic boundary condition of the form

$$T(0,t) = \langle T_0 \rangle + \Delta T_0 \sin(\Omega t). \quad (15)$$

Assuming a semi-infinite solid below, Carslaw and Jaeger (1959) show that the solution to (13) with this boundary condition has the form

$$T(z,t) = \langle T_0 \rangle + \Delta T_0 e^{-z/d} \sin(\Omega t - z/d), \quad (16)$$

where

$$d \equiv (2K/\Omega)^{1/2} = (2\lambda/(\rho c \Omega))^{1/2} \quad (17)$$

has the significance of a diurnal skin depth for the penetration of a thermal wave of period Ω applied at the surface. Thus, the typical thermal diffusivity for soil of $K = 5 \times 10^{-7} \text{ m}^2/\text{s}$ quoted earlier corresponds to a characteristic diurnal skin depth of some $d = 0.12 \text{ m}$ (12 cm).

The force-restore approximation is based on the fact that while the surface forcing terms of (14a) are not precisely periodic of a sinusoidal form, the influence of the solar forcing of period Ω establishes the penetration of diurnal waves in the manner of the exact solution, although the actual boundary condition on the heat flux of (14a) must be satisfied to transfer the proper amount of heat to and from the soil. Taking the partial derivative of (16) with respect to z and multiplying the result by $-d$ gives the relation,

$$-d(\partial T / \partial z) = \Delta T_0 e^{-z/d} (\cos(\Omega t - z/d) + \sin(\Omega t - z/d)).$$

Moreover, taking the partial t derivative of (16), multiplying the result by $1/\Omega$, adding T from equation (16) and subtracting $\langle T_g \rangle$ from both sides of the equation leads to the relation,

$$\frac{1}{\Omega} \frac{\partial T}{\partial t} + T - \langle T_g \rangle = \Delta T_0 e^{-z/d} (\cos(\Omega t - z/d) + \sin(\Omega t - z/d)).$$

Recognizing that the right-hand-sides of the foregoing two equations are identical, and that the heat flux through the soil at any depth is $G = -\lambda \partial T / \partial z$, we may eliminate the group of terms on the right-hand-sides to get

$$\frac{1}{\Omega} \frac{\partial T}{\partial t} + T - \langle T_g \rangle = -d \frac{\partial T}{\partial z} = -\frac{d}{\lambda} \left(-\lambda \frac{\partial T}{\partial z} \right) = \frac{d}{\lambda} G. \quad (18)$$

The force-restore ordinary differential equation for surface soil temperature is now obtained by evaluating (18) at the surface ($z = 0$, $T = T_g$) using the actual (nonsinusoidal, in general) boundary condition $G(0, t)$ from (14a):

$$\frac{dT_s}{dt} = \frac{2}{C_s}(S - R - F - L \cdot E) - \Omega(T_s - \langle T_s \rangle), \quad (19)$$

where the heat capacity per unit surface area appearing above,

$$C_s \equiv \rho c \cdot d = 2\lambda/(\Omega d) \quad (20)$$

corresponds to diurnal forcing cycles.

In the periodic solution $\langle T_s \rangle$ as defined in (15) is the mean soil surface temperature; in the force-restore equation (19) it was suggested by Blackadar (1976) that for short range projections $\langle T_s \rangle$ be treated as a constant whose value is estimated from the mean air temperature for the prior 24 hours. For forecasts of three days or more Deardorff (1978) suggests the variation of $\langle T_s \rangle$ be computed from

$$\frac{d\langle T_s \rangle}{dt} = \frac{1}{\langle C_s \rangle}(S - R - F - L \cdot E), \quad (21)$$

where $\langle C_s \rangle = (365)^{1/2} C_s$ is the heat capacity per unit surface area for the annual thermal wave.

In the sample calculations presented later, we are interested in diurnal cycles, so $\langle T_s \rangle$ is treated as a constant and C_s has the diurnal value. For the prior typical soil values of $\rho c = 2.1 \times 10^6 \text{ J/}^\circ\text{K-m}^3$ and $d = 0.12 \text{ m}$ the heat capacity per unit surface area is $C_s = 2.5 \times 10^5 \text{ J/}^\circ\text{K-m}^2$. Notice that in the force-restore formulation (as in the integral formulations of Arakawa (1972), etc.) the various thermal properties of soil are subsumed into the one parameter C_s . Notice that the last term in (19) acts to restore T_s exponentially to some mean (or deep) soil temperature if the surface forcing terms are removed. To use this differential equation in the present context we need to express the fluxes S , R , F and E in terms of the local values of T_s , u_1 , T_1 , q_1 and parameters characterizing the particular site.

Consider first that the sensible heat flux is expressible from (2), (10) and (11) as

$$F = -\rho C_p u^* T^* = -C_H \cdot \rho C_p u_1 (T_1 - T_s), \quad (22)$$

where

$$C_H = C_H(z_1/z_0, z_1/L) \equiv \kappa^2 / (\alpha_0 [\ln(z_1/z_0) - \psi_u(z_1/L)] [\ln(z_1/z_0) - \psi_T(z_1/L)])$$

is the surface layer's heat transfer coefficient referred to level z_1 . Since this flux is positive upwards we generally have $F > 0$ when $T_s > T_1$. Thus, knowing u_1 , T_1 , q_1 , T_s and q_s we would first find L from (9); the corresponding sensible heat flux follows immediately from (22).

Analogously, the latent heat flux carried by water evaporation or condensation at the surface is expressible from (2), (10), (11) and (12) as

$$LE = -\rho L u^* q^* = -C_H \cdot \rho L u_1 (q_1 - q_s). \quad (23)$$

In principle this can be handled similarly to the sensible heat flux although a major problem still exists insofar as we have not yet specified how to find the surface humidity q_s . A number of approaches exist, all of which require additional consideration of evaporation and evapotranspiration in the soil-vegetative component of the hydrological cycle. However, a fundamental idea in all of these methods is that the actual evaporative rate cannot exceed the potential evaporation rate which would obtain if the humidity at the surface were saturated at the value corresponding to T_s ,

$$E_o = -C_H \cdot \rho u_1 [q_1 - q_{sat}(T_s)], \quad (24)$$

where $q_{sat}(T_s)$ is calculable from the Clausius Clapeyron equation.

It is useful sometimes to represent the actual evaporation rate in terms of an actual-to-potential evaporation ratio $\beta \equiv E/E_o$. As discussed by Sellers (1965), it is helpful also to distinguish two stages of evaporation from soil and vegetation which depend on the volume fraction of soil moisture in the active soil layer W , usually expressed in millimeters of water. In the first stage, when the soil moisture content is greater than some critical value W_k , evapotranspiration proceeds at about the potential rate E_o , and depends mainly on external meteorological factors ($\beta = 1$ when $W \geq W_k$). In the second stage, when the soil moisture content is less than the critical value, the rate of evapotranspiration depends on the soil moisture content, with the relationship often assumed to be linear ($\beta = W/W_k$). Clearly, when the soil is below the saturation value W_k , it is necessary to either measure or model the evolution of W to predict evaporation. Deardorff (1977), for example, has proposed an extension of the force-restore approach as a prognostic equation for W driven by the difference between evaporation and precipitation rates and a restore term proportional to the difference between the local W and a long-term average $\langle W_s \rangle$. Thus an effective β can be computed simultaneously, or simply prescribed. Note that equating the actual evaporation rate $-C_H \cdot \rho u_1 (q_1 - q_s)$ to βE_o gives a relation for the surface humidity in terms of β , q_1 and T_s ,

$$q_s = (1-\beta)q_1 + \beta q_{sat}(T_s), \quad (q_s \leq q_{sat}). \quad (25)$$

In turn, this relation can be used to specify the surface specific humidity appearing in equations (9), (12) and (23).

Turning now to the solar radiation term S , we recognize first that diurnal variations in heating are associated with the daily variation in the solar zenith angle Z (the angle between a line pointing toward the sun and

a vertical line normal to the earth's surface at the latitude of interest -- e.g., when the sun is directly overhead $Z = 0^\circ$). It can be shown that the cosine of the zenith angle is expressible as

$$\cos Z(t) = \sin \phi \sin \delta + \cos \phi \cos \delta \cos(\Omega t), \quad (26)$$

where ϕ is the latitude, δ is the solar declination angle and Ωt is the hour angle relative to solar noon (i.e., $\Omega t = 0^\circ$ @ 1200 hr and increases by 15° every hour so that, for example, $\Omega t = -90^\circ$ @ 0600 hr). The solar declination varies slowly relative to the hour angle in a sinusoidal fashion with a period of one year and a maximum amplitude at the summer solstice (June 21) of $23^\circ 27'$ and minimum at the winter solstice of $-23^\circ 27'$. Values for each hour and day of the year may be obtained from *The Nautical Almanac* published by the U.S. Government Printing Office although for the present analysis we may justifiably take δ constant over a diurnal cycle. It should be clear, however, that the zenith angle only has significance during daylight hours when Z lies between 0° and $+90^\circ$. The hour angles (or times of day) corresponding to local sunrise or sunset are therefore found by setting $\cos Z = 0$ in (26) and solving for the hour angle,

$$\cos(\Omega t) = -\tan \phi \tan \delta.$$

The two possible values of Ωt between -90° and $+90^\circ$ correspond to local sunrise and sunset, respectively.

Under cloud-free conditions, the direct solar radiation absorbed by the ground may be written,

$$S(t) = S_0 \cos Z (1 - A) \tau^{\sec Z},$$

where S_0 is the solar constant, A is the surface albedo and τ is the atmospheric

transmission coefficient. The commonly accepted value of the solar constant (the frequency-integrated solar radiation flux per unit area falling on a plane perpendicular to the sun's rays at the top of the atmosphere) is $S_0 = 2.0 \text{ ly} \cdot \text{min}^{-1}$ ($2.0 \text{ cal} \cdot \text{min}^{-1} \cdot \text{cm}^{-2} \approx 1400 \text{ W} \cdot \text{m}^{-2}$) which we adopt here as well, for numerical calculations. Sellers (1965, p. 21) has tabulated a range of albedos, or reflectivities, of various surfaces in the shortwave portion of the electromagnetic spectrum (wavelengths less than $4.0 \mu\text{m}$, the region relevant to reflection of solar radiation). By definition, these albedos are between 0 and 1 and increase with increasing reflectivity; thus we have A in the range .05-.15 for coniferous forests, .1-.2 for deciduous forests and green meadows, .15-.20 for tundra, chaparral and wet-season savanna and .25-.30 for dry-season savanna and desert. In the present study we adopt a nominal value of $A = 0.12$ for purposes of initial calculations. For the transmission coefficient τ , values in the range of 0.75-0.90 are typical of the fraction of solar radiation penetrating to the surface under cloud-free overhead sun conditions. It is worth noting here again that the time dependence of $S(t)$ in the cloud-free case is dominated by the ωt diurnal periodicity of the zenith angle consistent with the assumption made earlier of an effective diurnal penetration depth for the solar heating wave.

A semi-empirical correction for solar radiation can be developed for overcast skies in terms of the degree-of-cloudiness parameter n often available with the surface observations. We note first that the cloud-free term $S^* = \cos \gamma \cos \theta$ represents the direct component of radiation from the sun through the atmosphere incident on the surface. Some fraction of this is diffusely scattered by the atmosphere and reaches the surface as well, although this is usually neglected or treated implicitly in the cloud-free case. The

fraction of diffuse to direct solar radiation $\epsilon = S_{diff}/S^*$ is solar zenith angle dependent varying from 5% for an overhead sun to 15% for the sun at the horizon. Assuming a simple linear variation with $\cos Z$ (Kahle, 1977) gives

$$\epsilon(Z) = 0.05 + 0.10(1 - \cos Z). \quad (27)$$

Now, under cloudy sky conditions, the direct component of radiation is reduced to $S^*(1 - n)$ as discussed by Kondratyev (1969, p.312), while the diffusive component becomes more important since it now includes the diffusive scattering by clouds. Kondratyev (1969, p. 399) writes a parameterized form of the diffuse radiation flux under cloudy conditions as $S^*[\epsilon(1 - n) + Kn(1 + \epsilon)]$, where K is an empirical latitude-dependent parameter representing the solar radiation transmitted by diffuse radiation through clouds. Based on Table 8.5 of Kondratyev (1969, p. 468), however, the variation of K with latitude is relatively weak in the middle latitudes, with numerical values in the range of 0.32 to 0.36 from $\phi = 0^\circ$ down to $\phi = 55^\circ$. For our calculations, the constant value $K = 0.34$ is adopted for the ratio of diffuse radiation transmitted through clouds to the direct component. Accordingly, the total solar flux incident vertically on the ground in the presence of clouds may be written $S^*[(1 - n) + \epsilon(1 - n) + Kn(1 + \epsilon)]$. Rearranging, and using the numerical value of K , we may write the solar flux actually absorbed by the ground in the form,

$$S(t, n) = S_0 \cos Z \tau^{8 \cos Z} (1 + \epsilon(Z))(1 - A)(1 - 0.66n), \quad (28)$$

where $\epsilon(Z)$ is given by (27) and $Z(t)$ by (26).

The remaining term on the right-hand-side of (19) to be parameterized is the terrestrial longwave (infrared) radiation R upwelling from the surface. Observations indicate that this flux is correlated under cloud-free conditions

with the ground surface temperature T_g and the water vapor pressure a few meters above the surface, say $e_1 = e(z_1)$, by expressions of the form $R = \sigma T_g^4 f(e_1)$, where σ is the Stefan-Boltzmann constant $= 8.14 \times 10^{-11} \text{ ly} \cdot \text{min}^{-1} \cdot ^\circ\text{K}^{-4}$ ($8.14 \times 10^{-11} \text{ cal} \cdot \text{cm}^{-2} \cdot \text{min}^{-1} \cdot ^\circ\text{K}^{-4} = 5.86 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot ^\circ\text{K}^{-4}$) and the function $f(e_1)$ is determined empirically. For the present model, we use the formulation for net upwelling surface infrared radiation in the cloud-free case proposed by Brunt and quoted by Sellers (1965, p. 53) in the form

$$R = \epsilon \sigma T_g^4 (1 - \alpha - b \sqrt{e_1}),$$

where ϵ , here, is the surface emissivity and α and b are empirical coefficients. Ordinarily the vapor pressure in this expression is expressed in mm Hg. This can be computed readily from the reference-level specific humidity q_1 (kg/kg) and the station pressure p (in. Hg) by

$$e_1 (\text{mm Hg}) = 1.61 q_1 \times p (\text{in. Hg}) \times 25.4 \text{ mm/in.} \quad (29)$$

Sellers (1965) in his Table 7 quotes surface emissivities for various natural and vegetative surfaces in the range of 0.9 to 0.95. For the coefficients in Brunt's formula we take Budyko's (1956) values, $\alpha = 0.61$ and $b = 0.050 (\text{mm Hg})^{-1/2}$. These give results similar to the values in Kondratyev's (1969) book and are close to the median of twenty-two evaluations quoted by Sellers (1965). In reality, however, the sky generally contains some cloudiness, and the infrared flux term should be corrected approximately to account for this effect. Kondratyev (1969, pp. 575-576) suggests an empirical correction of the form $R = R^*(1 - cn)$, where R^* is the cloud-free value of infrared radiation, $c = 0.76$ is an empirical coefficient, and n is degree-of-cloudiness. Notice that clouds have a blanket-

ing or insulating effect on ground temperature insofar as they reduce the radiant heat loss from the surface through back radiation. Combining the relations and numerical factors derived thus far gives the following parameterization for the terrestrial infrared term in (15),

$$R(T_g, q_1, n) = 0.92 \sigma T_g^4 (0.39 - 0.050 \sqrt{e_1}) (1 - 0.76n), \quad (30)$$

where e_1 is related to q_1 by (29).

4. Numerical Model and Sample Results

The foregoing scheme for generating surface layer turbulent fluxes from mean flow observations has been implemented in a FORTRAN computer code (TURBFLUX) which has been run thus far on the CDC 6600 machines at Brookhaven National Laboratory and the Courant Institute of Mathematical Sciences at New York University. The logic of the calculation is reviewed below.

The basic input data are the reference-level wind speed $u_1(t)$, temperature $T_1(t)$, relative humidity $r_1(t)$ and fractional sky cover $n(t)$ in digital form over the period of interest, the reference-level height z_1 , aerodynamic roughness z_0 , specific heat per unit surface area C_s , initial surface temperature $T_g(0)$, mean soil temperature $\langle T_g \rangle$, latitude ϕ , solar declination δ , surface albedo A , surface emissivity ϵ , ground wetness parameter β and integration timestep ΔT . At the initial time, and at all subsequent timesteps after T_g is computed, the following routines are executed:

- (1) Compute q_1 and q_g from r_1 , T_1 , T_g , β and equation (25).
- (2) Compute T_{v1} and T_{vg} and evaluate Richardson number, $Ri_1 = g z_1 (T_{v1} - T_{vg}) / (T_g u_1^2)$.
- (3) If decoupled ($0.21 \leq Ri_1$) set $u^* = T^* = q^* = 0$. If stable ($0 < Ri_1 < 0.21$) find L from quadratic equation, If unstable ($Ri_1 < 0$) find L from Newton-Raphson

iteration solution to equation (9).

- (4) From u_1 , T_1 , q_1 , T_g and q_g find u^* , T^* and q^* for stable, neutral or stable cases from equations (10), (11) and (12) and Table 1 functions.
- (5) Compute F from u^* , T^* and equation (22).
- (6) Compute $L \cdot E$ from u^* , q^* and equation (23).
- (7) Compute S from t , n and equation (28).
- (8) Compute R from T_g , q_1 and n from equation (30).

These operations completely define the right-hand-side of the force-restore equation (19) at each timestep. To improve the accuracy of the integration a semi-implicit technique is used to evaluate $T_g(t)$ numerically from (19) in the TURBFLUX code. For the calculations discussed next an integration time of $\Delta t = 10$ min (600 s) was used with outputs printed every hour.

The first case studied was for constant atmospheric forcing, dry soil, mid-latitude equinox conditions for the soil and atmospheric parameters given in the caption of Figure 1. Here, the reference-level windspeed and temperature were held constant at $u_1 = 4$ m/s and $T_1 = 280$ °K over a diurnal cycle. As shown in Figure 1(a) the friction velocity varies slightly about its neutral value, being somewhat higher during the unstable daytime phase and lower at night. The transition from unstable to stable surface layer flow is marked by the sign change in the buoyant temperature scale T^* and occurs slightly after sunset, with another transition back to unstable turbulent flow the following morning. This relatively smooth variation in the buoyant stability of the surface layer can also be traced in the variation of the reciprocal Monin-Obukhov length shown in Figure 1(b). Also shown is the computed variation of soil surface temperature. Notice the rapid drop in the afternoon as the solar radiation diminishes, followed by a somewhat slower radiational cooling at night with a subsequent build-up the following day.

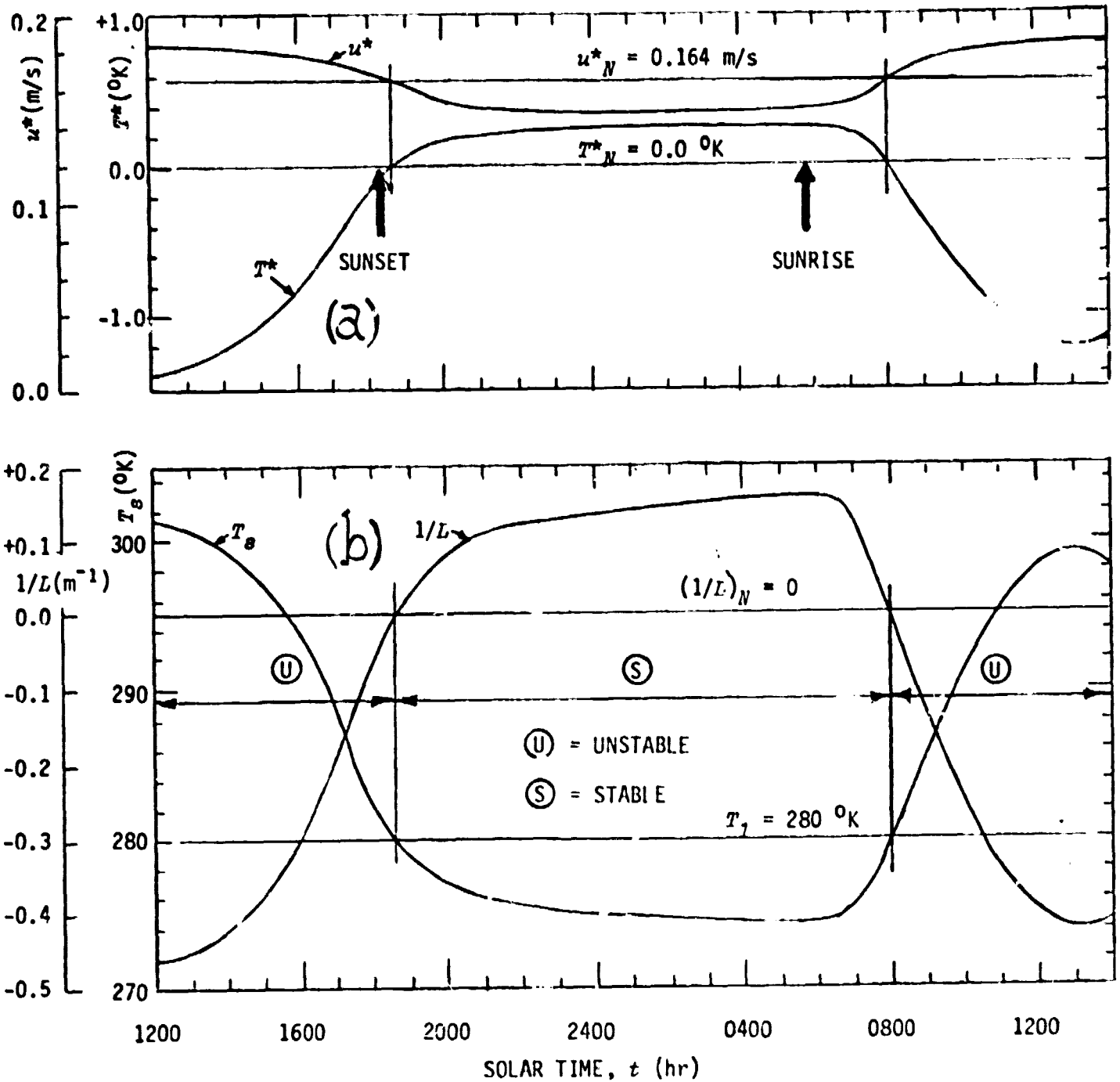


FIGURE 1 Typical diurnal cycle of atmospheric surface layer with constant atmospheric forcing ($@ z_1 = 1.5$ m; $u_1 = 4$ m·s⁻¹, $T_1 = 280.0$ °K) and dry soil ($\beta = 0$) computed from present model for the soil and atmospheric parameters: $C_s = 1.6 \times 10^5$ J·m⁻²·°K⁻¹, $z_0 = 4 \times 10^{-4}$ m, $\langle T_s \rangle = 282.0$ °K, $\phi = 45$ deg, $\delta = 0$ deg, $A = 0.25$, $\epsilon = 0.90$ and $\tau = 0.85$. (a) Friction velocity u^* and buoyant stability temperature scale T^* and (b) soil surface temperature T_s and reciprocal Monin-Obukhov length $1/L$.

In practice, the atmospheric reference-level properties are likely to vary significantly over a diurnal cycle. Accordingly, we chose as our initial test of variable atmospheric forcing to model the observational conditions of Kahle (1977) who measured both reference-level windspeed and temperature and soil surface temperature (by radiometer) at sites in the Pisgah Crater-Lavie Lake region of the Mohave desert in California. This is an arid, vegetatively sparse region whose surface includes both basalt and clay playa zones. Soil moisture measurements by Kahle (1977) indicated that latent heat transfer could be ignored in the surface energy balance.

The variable reference-level winds and temperatures used in our simulation of this case are shown in Figure 2, as are the other input parameters which are given in the caption. Shown in Figure 2(b) is the model-computed surface temperature compared with the radiometer data, where the error bars on the observations correspond to a range of values for the region. The parameter values of latitude, solar declination, albedo, surface emissivity and transmissivity used in the model are those given by Kahle (1977) and the surface roughness was chosen to recover the heat transfer coefficient used in Kahle's (1977) model under neutral conditions. The value of C_g was adjusted however to obtain the solid curve, a reasonable procedure in view of the semi-empirical nature of this parameter. The complex variations predicted for the surface layer with this forcing are, however, better revealed in Figure 3. The variability of u^* and w^* shown in Figure 3(a) are for this case the result of variable atmospheric forcing at the reference-level as well as variations in stability associated with surface temperature changes. In addition, the variation of Richardson number and Monin-Obukhov length for this case are considerably more complex than for the constant-forcing case.

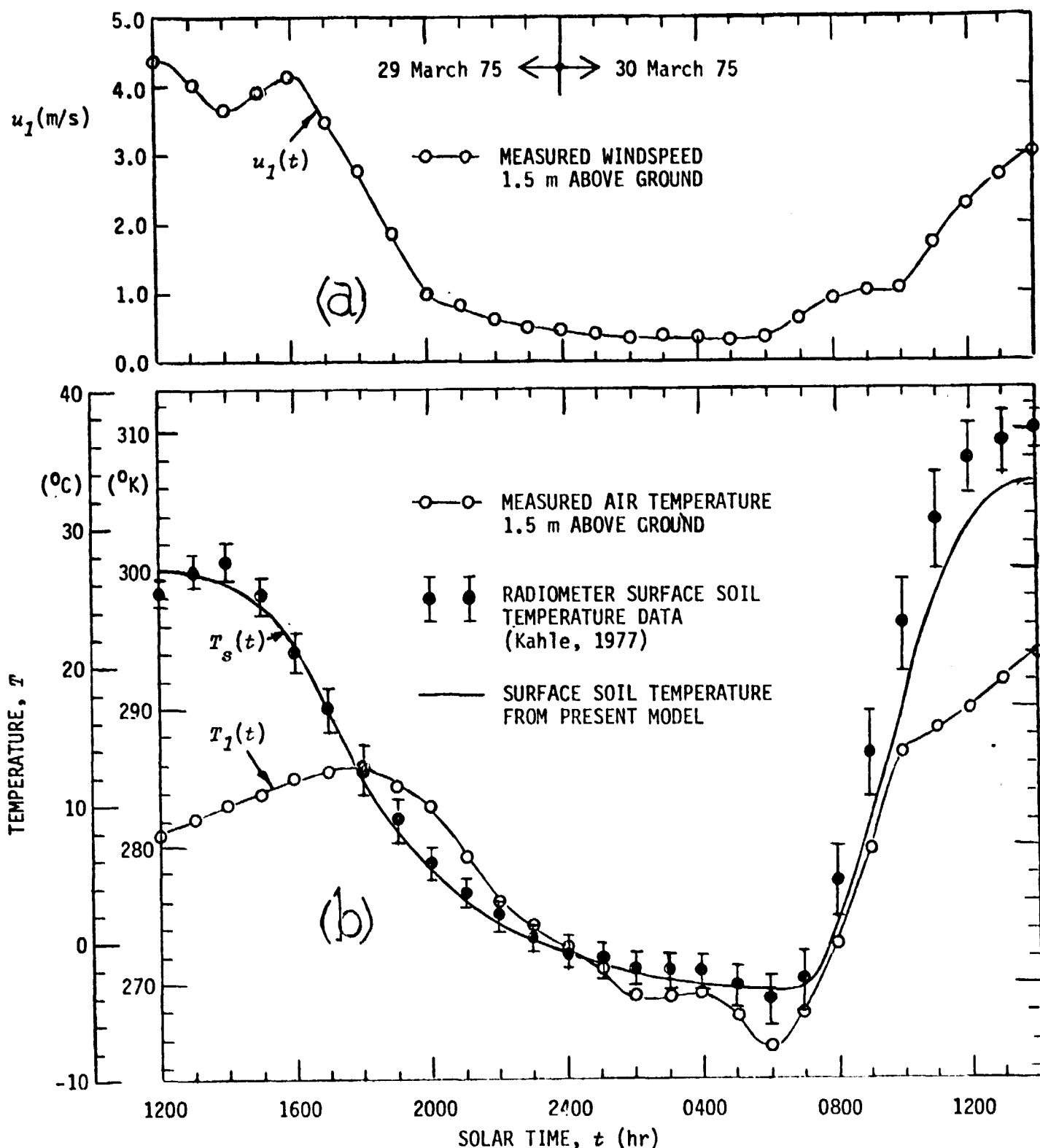


FIGURE 2 Diurnal cycle of atmospheric surface layer with variable atmospheric forcing ($@ z_1 = 1.5$ m; $u_1 = u_1(t)$, $T_1 = T_1(t)$) and dry soil ($\beta \approx 0$) in the Pisgah Crater-Lavac Lake region of the Mohave desert in California including model results for the soil and atmospheric parameters: $C_s = 2.5 \times 10^5 \text{ J} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$, $z_o = 3 \times 10^{-4} \text{ m}$, $\langle T_g \rangle = 301.16 \text{ }^{\circ}\text{K}$ (28 $^{\circ}\text{C}$), $\phi = 34.65 \text{ deg}$, $\delta = 3.1 \text{ deg}$, $\lambda = 0.44$, $\epsilon = 1.00$ and $\tau = 0.80$. (a) Measured windspeed at reference level and (b) Measured air temperature at reference level and comparison of measured and computed surface soil temperatures.

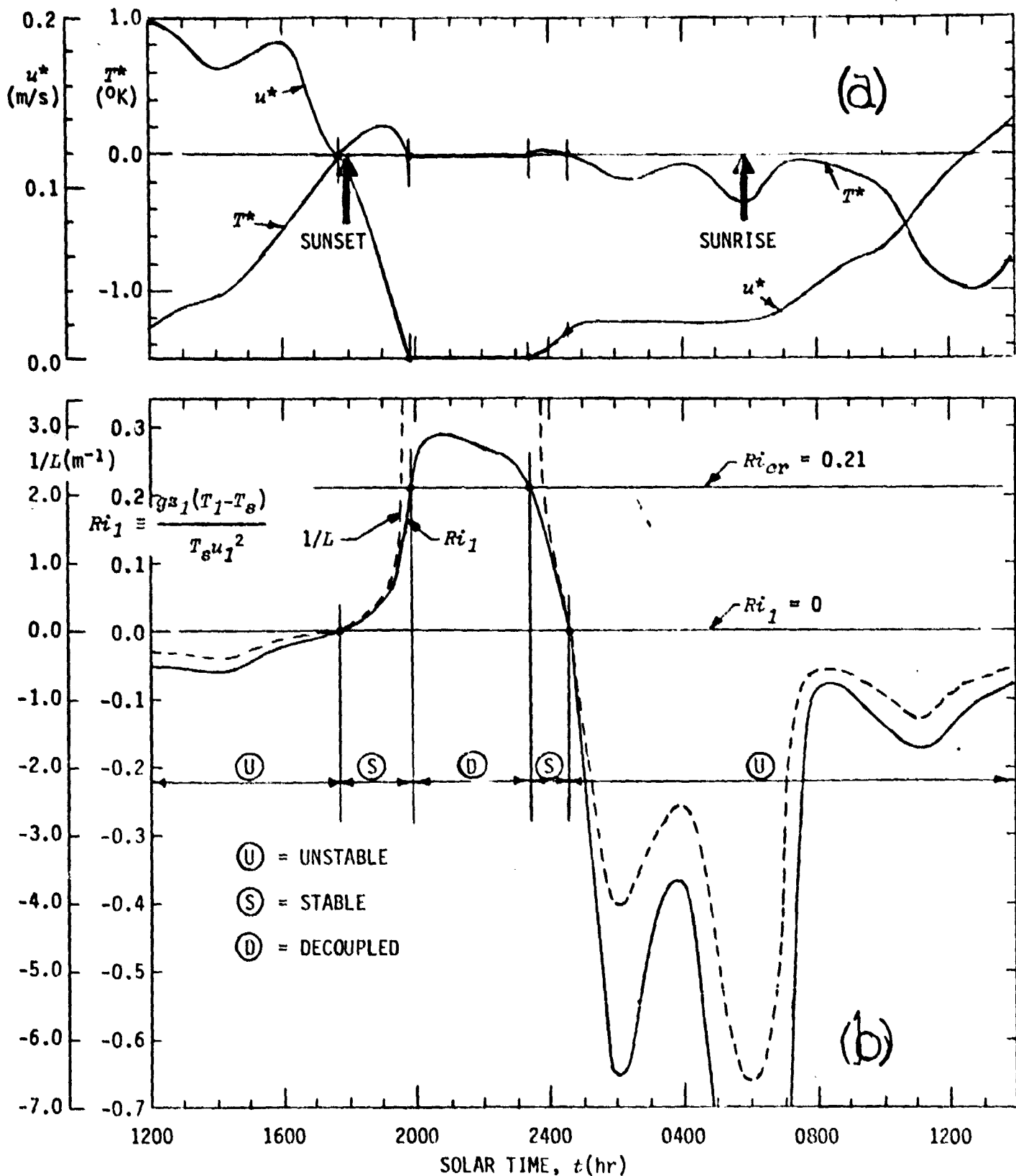


FIGURE 3 Diurnal cycle of atmospheric surface layer properties computed with present model for variable atmospheric forcing and parameters of FIGURE 2. (a) Friction velocity u^* and buoyant stability temperature scale T^* and (b) reciprocal Monin-Obukhov length $1/L$ and reference-level Richardson number Ri_1 .

As indicated in Figure 3(b) the computed evolution of Richardson number includes an unstable region until approximately sundown near 1800 hr, followed by an increasingly stable zone which becomes decoupled at about 2000 hr and reattaches as a stable layer around 2300 hr, followed by a transition back to unstable conditions which persist into the next day. Notice that the "unusual" transition to unstable flow in the nocturnal phase corresponds to the drop in air temperature below the surface temperature slightly after midnight, while the large negative values of Richardson number and reciprocal Monin-Obukhov length in the early morning of 30 March correspond to the low windspeeds at these times. Of particular interest is the model's ability to predict decoupling of the stable layer, which would not be possible with a constant heat transfer coefficient.

5. Concluding Remarks

The scheme documented here for finding the turbulent fluxes at the bottom of the planetary boundary layer from measurement of mean flow properties at some reference height in the range of 1-10 meters above the surface has been tested with reasonable success against surface temperature data over a diurnal cycle. The method is based on mating Monin-Obukhov similarity theory with the force restore formulation of the ground temperature equation developed originally by Bhumralkar (1975) and Blackadar (1976). Indeed the present model parallels parts of Blackadar's (1976) model for the nocturnal boundary layer, although his reference level is driven by prognostic equations rather than direct observations. In view of the current interest in understanding and modeling the decoupling of very stable layers at night, it would be interesting to test the model's ability to predict such decoupling in a controlled observational situation where decoupling actually is measured.

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